Digital Signal Processing SS 2019/20

Exercise Sheet 1

Due date: 1.5.2019

Exercise 1

A complex number can be represented in classic cartesian, polar, vector, and matrix form as follows:

$$z = a + \mathrm{j}b, \quad z = r\mathrm{e}^{\mathrm{j}\theta}, \quad z = \begin{pmatrix} a \\ b \end{pmatrix}, \quad z = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}.$$

Rewrite the matrix form using the polar coordinates r, θ of z in place of the cartesian coordinates a, b. How can one compute the product of two complex numbers using the matrix form? Express the inverse $\frac{1}{z}$ using each representation.

Exercise 2

Calculate

$$z = \frac{2 - \mathrm{j}3}{5 + \mathrm{j}12}$$

in classical, point, polar, and matrix form.

Exercise 3

Simplify the following complex terms and give the result in both cartesian and polar form.

a) $(1 - \mathrm{j})^{43}$

b) $2\mathrm{e}^{-32\pi\mathrm{j}/3}$

c) $3\mathrm{e}^{\mathrm{j}\pi/3} + 4\mathrm{e}^{-\mathrm{j}\pi/6}$

d) $\frac{z-1}{z+1}$, $z \in \mathbb{C} \setminus \{-1\}$.

Exercise 4

Solve the following equations for $z \in \mathbb{C}$ and check the solution with the Matlab-function `roots`.

a) $z^2 + 2z + 2 = 0$,

b) $z^2 + 2\mathrm{j}z = 1$,

c) $z^n = 1 - \mathrm{j}$.

Exercise 5

Plot the following shapes in one figure using the Matlab `plot` command with complex numbers as arguments.

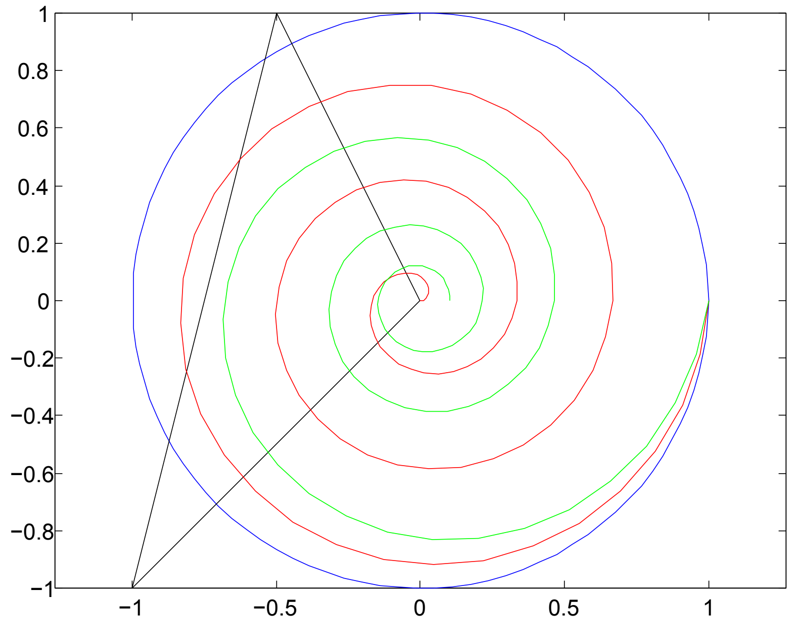
a) Plot a blue unit circle.

b) Plot a black triangle that visualizes the addition $z_1+z_2 = z_3$ with $z_1 = -1-\mathrm{j}$ and $z_2 = 0.5+2\mathrm{j}$.

c) Plot a red spiral starting at the origin. The distance d to the origin grows linearly with the angle and has 3 rotations within the unit circle.

d) Plot a green spiral. Now d grows exponentially with the angle, has again 3 rotations and starts at $\frac{1}{10} + \mathrm{j}0$.

The result should look like this:



Exercise 6

a) Give the 2D-matrix that represents a rotation by an angle ϕ. Multiply this matrix with another matrix that represents an rotation by an angle ψ. Find 2 trignometric identities from the result.

b) Prove the trignometric identities again using complex calculus.

c) Show, that $\cos\phi = \frac{1}{2}\left(e^{\mathrm{j}\phi} + e^{-\mathrm{j}\phi}\right)$ Find a similar expression for $\sin\phi$.